**Will Quadratic Voting Produce Optimal Public Policy?**

**by**

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**and**

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Abstract

Under quadratic voting, people are able to buy votes with money and the cost of votes increases at an increasing rate. If people are rational they will buy votes until the marginal cost of a vote is equal to the marginal benefit, which is the value of the election to the purchaser weighted by the probability that the last vote purchased will determine the election.

The Coase Theorem applies here. If there are no costs of information and no cost of voting, the subjectively assessed probability should be the same for all voters. We show that the political outcome in this case will be optimal.

If information is costly, however, small differences in perceived probabilities can lead to large welfare losses. Suppose the group that favors more spending on a project estimates its probability at one-in-a-million, on the average, while the opponents’ estimate is two-in-one-million. These two numbers are so small, they seem almost equivalent. Yet what matters in equilibrium is not their absolute size but their ratio. A two to one ratio can produce a misallocation of public sector resources far greater than anything we expect to see in the private sector.

Large welfare losses can also be generated by the costs of voting (time, transportation etc.), by collusion among voters and by the efforts of organized groups, political candidates and political parties to induce voting behavior that is otherwise not in people’s rational self-interest. Ordinary voting (without vote buying) also leads to non-optimal results. But under certain conditions, vote buying could make things worse.

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I. Introduction

 Quadratic voting has been proposed by several scholars, including a new book by Posner and Weyl (2018). It belongs to a class of voting models that allows voters to express their intensity of preference as opposed to one-person-one-vote. Perhaps the most popular of these is the probabilistic voting model in which voters are viewed by candidates as having a probability of voting that is responsive to changes in the platforms they propose. Analogously, Gary Becker (1983) considers pressure among interest groups as shaping political outcomes.

In a series of papers Goodman and Porter (1985, 1988, 2004) summarize the many ways one could influence an election – casting a vote, making campaign contributions, influencing others, etc. – as a homogeneous, continuous variable. If certain concavity/convexity conditions hold, there will be a unique platform that can defeat all others in a majority vote and the equilibrium is stable (Goodman, 1980). By treating the effort per unit of benefit (effort/benefit ratio) as the “price” voters are willing to pay to secure a dollar of benefit we were able to identify a host of equilibrium and comparative static properties of the model.

For purposes of this paper one finding is critical: In equilibrium, the marginal social benefit from a policy variable divided by its marginal social cost must equal the political price people are willing to pay for a dollar of benefit divided by the political price people are willing to pay to avoid a dollar of cost. Optimality requires that the prices be equal. In voting models, a sufficient condition for Pareto optimality is that the effort/benefit ratio be equal for all voters (1985). For opposing interest groups, the aggregate effort/benefit ratio must be equal for the two groups (2004). Quadratic voting is said to mimic this condition and it is claimed to promote efficiency relative to other voting mechanisms (Lalley and Weyl, 2014).

Although it may not have been their original intention, Lalley and Weyl found an ingenious way of showing us how the Coase theorem can be applied to politics. If there are no information costs, then everyone should have the same estimate that a single vote cast will determine the outcome of an election. If, in addition, there are no costs of voting, the effort/benefit ratio of all voters will tend to be the same.

Although Becker, Goodman and Porter did not investigate this issue, it seems likely that with a few assumptions zero transactions costs would produce optimal outcomes in those models as well. However, just as transactions costs may cause markets to fail to perform efficiently, they may also cause political systems to be sub-optimal.

One area where quadratic voting may produce efficient outcomes is in the market for corporate governance. Posner and Weyl (2013) propose the use of quadratic voting in corporations as a better means to control managerial shirking, to minimize the impact of asymmetric information among stock holders, and to discourage exploitation by majority shareholders. The efficient market hypothesis made popular by Eugene Fama (1965, 1970) asserts that these inefficiencies are reflected in the corporation’s stock price and that votes purchased with shares of stock and voted linearly would promote efficiency. Of course, the efficient markets hypothesis fails when information is not properly evaluated and when investors are over-exuberant (Shiller (2005) causing the problems Posner and Weyl address using quadratic voting. The conditions that weaken the efficient markets hypothesis also reduce the efficiency characteristics of the quadratic voting model. Quadratic voting may work better in the area of corporate governance because these conditions are less severe there than in other voting environments.

In the more general case, however, the claim that quadratic voting promotes efficiency rests on the assumption that individual voters know nothing about the intent of other voters. While this is a common assumption about private market decisions, political decisions are driven by groups organized around common interests who share information and encourage their members to vote. In the political environment, systematic differences in information and voter participation across opponent and proponent groups is almost certain.

Moreover, Goodman and Porter (2004) show that even small deviations in aggregate effort/benefit ratios will lead to huge welfare losses. Other features of voting systems are likely to amplify these problems.

II. Quadratic Voting

In quadratic voting one can buy votes (and presumably fractional votes) so that voting is on a continuum. To normalize, assume one can buy n votes for *$n2* for or against an issue or candidate and the most votes wins. The marginal cost of *n* votes is *2n*. The expected benefit is *pB* where *p* is the perceived probability that the nth vote will break a tie (admittedly very small) and *B* is the benefit to the vote buyer of a favorable political outcome. So, if people are rational maximizers,

(1) *pB* = *2n* or *p/2* = *n/B* for each voter,

where *n/B* is the effort/benefit ratio.

If the subjective probability, *p*, is the same for everyone, the effort/benefit ratios will also be the same. Pareto optimality will be guaranteed. Suppose that there is a single issue (yes/no) election. Then if for “yes” voters is greater than for “no” voters, the “yes” voters must produce more votes. The side that places the greatest value on the issue is the side that wins.

Note that in equation (1) every voter is understating by a factor of *1-p* the true value of the election to herself. Unlike the marketplace, in the political system people are never expected to fully reveal their preferences. We can still get optimal outcomes however, if every voter understates her preferences to the same extent.

Note also that the probability of being the decisive voter quickly approaches zero as the number of voters becomes very large. Even so, small differences in the perception of these probabilities can cause large welfare losses. Suppose the beneficiaries of a public spending project believe their probability of being the decisive voter is 1/1,000,000, on the average, while taxpayers who must bear the cost believe their probability to be 2/1,000,000. Equation (1) shows that the opposition group will buy twice as many votes per unit of cost as the proponent group buys per unit of benefit, defeating any issue for which the benefit is less than twice the cost.

III. Inefficiency in Political Markets

            Harberger (1954) calculated the welfare loss from monopoly in the private sector to be about 1/10th of one percent of private sector output.  We use the same assumptions and methods of calculation here to measure the welfare consequences of misallocation in the public sector under a variety of voting models. Further, instead of a yes/no vote we allow a continuum of options with candidates able to take positions along the continuum.

            Let *B(Q)* be the benefit of the group advocating for more public-sector output, *Q*, and let *C(Q)* be the cost to the group opposing more output.  If there is a unique winning platform, *Qe*, it must satisfy

(2) .

Here, the superscripts, *p* and *o*, represent the proponents and opponents of spending and *MB* and *MC* represent marginal benefit and marginal cost of *Q* for the two groups, respectively. The *λs* in equation (2) represent group effort per unit of benefit in the Goodman Porter model. This is analogous to votes per unit of benefit in the quadratic voting model and group pressure in the Becker model.

Equation (2) says that in equilibrium, the marginal support the proponents offer for increasing the level of *Q* is exactly equal to the marginal loss of support from the opposition. If the *λs* are the same, marginal social benefit will equal marginal social cost and *Qe* will be the optimal level of output.

What if the *λs* are not the same? Harberger assumed constant marginal cost and unitarily elastic demand and approximated the welfare loss by

(3) ,

where *Q\** is optimal output and *Qe* is output determined by the political process.

In Figure 1 we assume that The welfare loss from political misallocation as a fraction of spending, *MCxQe*, can be expressed as a fraction of the effort/benefit ratios. In political equilibrium and, by the assumption of unitary elasticity, . Substituting these into equation (3) and dividing by spending reveals that the welfare loss per dollar of spending is where .

Thus, if the effort/benefit ratios differ by a factor of only two, the welfare loss is $.50 per dollar of public spending. If they differ by a factor of three, there will be a dollar of waste for every dollar spent.

IV. Reasons for Inefficient Outcomes[[1]](#footnote-2)

Costs of Information. Why would voters have different estimates of the probability that their vote will be election determining? One reason is that their knowledge is not the same and the cost of obtaining information is not free. Organized groups (see below) can provide their members with better information – and also misinformation (exaggerating, for example, the importance of their vote). A more serious problem is that people often do not have accurate information about the true benefit and the true cost to them of a policy change. Even with identical effort/benefit ratios, if the underestimate of the benefits of a project to those who receive them is twice the size of the underestimate of the costs to those who bear them, we will (again) have 50 cents of waste for every $1 of spending. If the underestimates differ by three-to-one we will (again) have a dollar of waste for every dollar of spending.

 The Cost of Voting. Consider a group of 200 voters who each stand to gain $1 if a particular proposal is passed and one individual who stands to lose $100. For the moment, assume that all 201 voters agree that the probability of being the decisive voter is 1/25. If voting is costless, the 200 voters whose expected benefit is $.04 each buy a .02 vote and the proposal receives 4.0 votes in favor. The one individual opposed to the issue has an expected value of $4.00 and buys 2.0 votes (at a cost of $4.00). The issue efficiently passes 4.0 to 2.0. However, if the cost of voting is $1.00, not one of the 200 proponents will vote while the single opponent will buy a $1.98 vote in opposition and the issue will inefficiently fail.[[2]](#footnote-3)

If voting is costly, one-person-one-vote systems favor concentrated interests over widely dispersed interests. However, quadratic voting intensifies that distortion.

Collusion. Assume there are four members in the opposition group: one who stands to lose $100 if the proposal passes and three others who would lose $1.00 if the proposal passed. If there are no voting costs and all perceive that the probability of being the decisive voter is 1/25, the proponent group buys 200 .02 votes (4.0 vote total). Working independently, the four opponents would by one 2.0 vote (for $4.00) and three .02 votes. The issue would efficiently pass 4.0 to 2.06.

 However, the opponent group can do better. The $4.00 that was to be spent on 2.0 votes could be distributed among the four opponents to spend on votes. Each $1.00 would buy 1.0 votes for each opponent. Coupled with the .02 votes the three opponents would purchase, the opposition would now tally 4.06 opposing votes and the issue would inefficiently fail.

 For any amount of money *M* that an interest group wishes to spend in total without collusion the optimal distribution of *M* among *N* members with collusion is *M/N* each.[[3]](#footnote-4) Moreover, the benefit of collusion increases with increases in variation in the amount each member of the coalition is willing to contribute to voting.[[4]](#footnote-5)

 Note that redistribution among those with common political interests is prevalent, even under one-person-one-vote. An employer may give workers time off (thereby paying them to vote) to go vote against an issue or candidate that is opposed by the company. In trade associations and PACs, those members with larger stakes tend to contribute more. However, quadratic voting would appear to increase the return from political organization and collusive activity.

Even with no transactions costs, perfect information and no costs of voting, there is a problem here that arises merely because the cost of buying votes is non-linear. With linear voting (constant cost of vote buying) the results would be optimal.

Costs of Collective Action: Special Interests. If there are costs of voting, Downs’ Paradox of Voting applies. For most people, voting never pays. The role of trade associations, PACs, and other activist groups is to persuade people to do what they otherwise would not do on their own. In elections, however, the outcome is a public good for those who support it and a public bad for those who are opposed. As such, one expects there to be free riders among voters. Goodman and Porter (2004) explicitly recognize this and point to differences in the ability of interest groups to overcome free riding as the reason to suspect different effort/benefit ratios for groups.

 Groups will play another role in quadratic voting when private interests are sufficiently high and voting is rational. Quadratic voting requires information about the probability of being the decisive voter and groups, with superior information about the voting behavior of others, are the likely source of such information.

Groups that are better organized, have better channels of communication, and, perhaps more importantly, have a means to punish free-riding behavior (Olson, 1965) will generate more votes per unit of benefit than other groups. Quadratic voting would appear to increase the return from such behavior.

Costs of Collective Action: Candidates and Political Parties. Political candidates and political parties spend a great deal of money on get-out-the-vote efforts. However (and this is a conjecture), there must be large diminishing returns. The reason: no matter how much you spend – even on those with a high probability of voting – the most you can get in return is one vote. And as candidates and parties get more dollars we presume that they chase after increasingly marginal voters. With quadratic voting, however, you can get more than one vote out of likely voters. You can get a second vote, and a third and a fourth. Instead of chasing after marginal voters you can increase your investment in likely voters and get them to vote many times.

It seems highly likely that quadratic voting would increase the marginal product of money in politics and, for better or for worse, lead to more of it.

V. Conclusion

 The recent literature on quadratic voting has served an important function. It draws our attention to how difficult it is to get optimal public policy, even if people can buy and sell votes. Quadratic voting is unlikely to ever be implemented. But the focus on it has increased our understanding of “government failure,” which should always be contrasted with market failure in deciding on the best role for government in the economy.

References

Gary S. Becker, “A Theory of Competition among Pressure Groups for Political Influence,” Quarterly Journal of Economics, Vol. 98 (1983), pp. 371-400.

Eugene Fama, "The Behavior of Stock Market Prices," Journal of Business 38 (1965), pp. 34–105.

\_\_\_\_\_\_\_\_\_\_\_\_, "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance 25 (2) (1970), pp. 383–417.

John C. Goodman, “A Note on Equilibrium Points of N-Person Concave Games,” Econometrica, Vol. 48, No. 251, 1980.

John C. Goodman and Philip K. Porter, “Majority Voting and Pareto Optimality,” Public Choice, Vol. 46, No. 2 (1985), pp. 173-86

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, “A Theory of Competitive Regulatory Equilibrium,” Public Choice, Vol. 59, No. 1 (1988), pp. 51-66.

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, “Political Equilibrium and the Provision of Public Goods,” Public Choice, Vol. 120, Nos. 3-4 (2004), pp. 247-266.

Steven P. Lalley and E. Glen Weyl, “Quadratic Voting,” Social Science Research Network (2015) <http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2003531>

Eric A. Posner and E. Glen Weyl, “Quadratic Voting as Efficient Corporate Governance,” The University of Chicago, Institute for Law and Economics (2013) <http://www.law.uchicago.edu/Lawecon/index.html>

­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, Radical Markets: Uprooting Capitalism and Democracy for a Just Society (Princeton: Princeton University Press, 2018).

Mancur Olson, The Logic of Collective Action, Cambridge: Harvard University Press (1965).

Robert J. Shiller, *Irrational Exuberance*, 2ND edition (2005). Princeton University Press

Figure 1: Political Distortions and Welfare Loss

λPMB

λOMC

WL

Marginal Cost (MC)

Marginal Benefit (MB)

 Qe

Value

 Q\*

Quantity

1. Lalley and Weyl (2015) address voting costs, collusion and different perceptions of the probability of casting the decisive vote. Their focus is on preserving the efficiency properties of quadratic voting and they demonstrate the robustness of quadratic voting over a wide range of assumption about the distribution of voters, their preferences and their perceptions of being the decisive voter. We have the opposite focus. We draw attention to the same phenomenon that leads to special interest dominance in voting models: concentrated versus dispersed benefits, collusion with heterogeneous versus homogeneous preferences and persuasion. [↑](#footnote-ref-2)
2. If the cost of voting is a sunk cost, perhaps the cost of going to the polls, the opponent, having borne the sunk cost, will buy a $2.00 opposition vote. The 200 proponents will not make the sunk cost investment. [↑](#footnote-ref-3)
3. *Xi = M/N* maximizes $\sum\_{i=1}^{N}X\_{i}^{\frac{1}{2}}$ subject to $M=\sum\_{i=1}^{N}X\_{i}$ for all i. [↑](#footnote-ref-4)
4. Let *m = M/N* be the optimal allocation of M among an interest group and consider any unequal distribution *m + δ* and *m - δ* *(0 < δ < m)* between two members. Collusive allocation *m* yields $2\sqrt{m}$ votes for the two individuals and the alternate allocation yields $\sqrt{m+δ}+\sqrt{m-δ}$ votes. Collusion then gains $G=2\sqrt{m}-\sqrt{m+δ}-\sqrt{m-δ}>0$ votes and the amount of votes gained by collusion increases with increases in variation, *δ*, as$ ∂G/∂δ=2δ(m^{2}-δ^{2})^{-\frac{1}{2}}>0$. [↑](#footnote-ref-5)