Is Income Always Good?

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Washington, D.C., or at least a large part of it, is something akin to a company town: Even those who are not employed directly by the federal government tend to have incomes that rise and fall with the salaries of government workers.

So the wages of federal workers are a frequent topic of conversation in D.C. Another frequent topic of conversation is housing prices. And there is an acute interest in the relation between the two. When I lived in Washington, I frequently heard the assertion that any raise for federal workers would drive up the price of housing, leaving Washingtonians no better off than before.

Of course the homeowners were less inclined than others to make this argument. So let us pose the question from the non-homeowner's point of view: Imagine a small company town in which everybody is identical, and the housing stock is fixed and owned by an absentee landlord who sets a market clearing rental price. Can an exogenous increase in income make the residents of such a town worse off?

The answer turns out to be yes if and only if housing is a Giffen good, which is to say that for practical purposes the answer turns out to be no. But it turns out to be no in a curious way.

Denote by ϵ_u the uncompensated price elasticity for housing (that is, ϵ_u is the elasticity of the uncompensated demand curve) and by ϵ_c the compensated price elasticity. Then we will show that the elasticity of net benefit to residents, with respect to exogenous increases in income, is given by the remarkably simple formula*

$$\frac{\epsilon_c}{\epsilon_u}. \tag{1}$$

* Throughout this discussion, we will treat all income and price elasticities as constant over the relevant ranges. Of course, budget constraints must invalidate this assumption when the changes are large enough. This in turn means that our conclusions will not be able to be taken completely seriously for certain parameter values. As equation (1) might suggest, the suspect cases are those in which ϵ_u is close to zero.

In particular, since ϵ_c is always negative, an increase in income yields a positive net benefit whenever ϵ_u is negative, and a negative net benefit when ϵ_u is positive, which is the Giffen case.

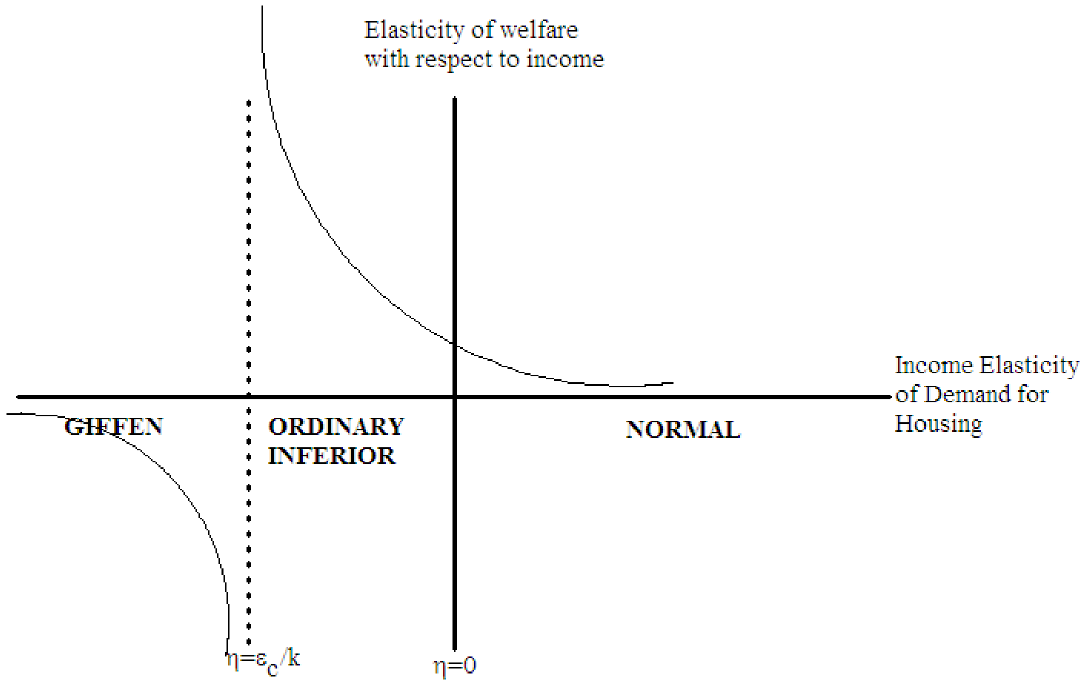
In view of the Slutsky equation

$$\epsilon_u = \epsilon_c - k \cdot \eta \tag{2}$$

(where k is the fraction of income spent on housing and η is the income elasticity of demand for housing), formula (1) can be rewritten as

$$\frac{\epsilon_c}{\epsilon_c - k \cdot \eta}.$$

Holding ϵ_c and k fixed and letting η vary, we get the following graph depicting the relationship between η and the income elasticity of net benefit:



The region to the left of the vertical asymptote is precisely the range of η that makes housing a Giffen good. If η falls between the asymptote and the vertical axis, then housing is inferior but not Giffen, and if it falls to the right of the axis then housing is a normal good.

Except in the Giffen case, exogenous income is always good thing—though it becomes less of a good thing as the income elasticity for housing increases. If housing is an inferior good, things are even better, provided we don't enter the Giffen range. A bit surprisingly, even though the Giffen case is the bad case, it is still true that if housing *must* be a Giffen good, then "the more Giffen the better". That is, it is bad for η to be less than ϵ_c/k, but better for it to be far less than just a little less.

The case where η is very close to ϵ_c/k is the hairtrigger one: If η is just slightly above this value, then exogenous income yields enormous gains; if it is just slightly below, then exogenous income yields enormous losses. (But the arbitrarily large gains and losses suggested by the asymptotic behavior should be taken with a grain of salt in light of footnote 1.)

Now we will give a proof of formula (1). Suppose that income increases by 1% and that this leads to a $t\%$ increase in housing prices. The quantity of housing demanded will increase by $\eta\%$ on account of the income increase and $t \cdot \epsilon_u\%$ (which is negative in the normal case) on account of the price increase, for a total increase of $(\eta + t \cdot \epsilon_u)\%$. But in equilibrium the quantity of housing demanded cannot change, so we get $\eta + t \cdot \epsilon_u = 0$ or

$$t = -\frac{\eta}{\epsilon_u}. \tag{3}$$

The percentage change in income spent on housing is equal to the percentage change in housing prices times the fraction of income spent on housing, or $t \cdot k$. So the percentage increase in income available to spend on other things is equal to the original 1% increase in income, minus $k \cdot t$. In view of equation (3), this is

$$\begin{aligned} 1 - t \cdot k &= 1 + k \cdot \frac{\eta}{\epsilon_u} \\ &= \frac{\epsilon_u + k \cdot \eta}{\epsilon_c}, \end{aligned}$$

which, in view of equation (2), is equal to expression (1).

There is also a nice geometric proof, using indifference curves, which the reader might prefer to discover for himself.