

When n identical heat reservoirs of heat capacity C are brought together and allowed to reach a final temperature T , the change in entropy is

$$Cn \log \left(\frac{T}{\text{Geometric Mean of Original Temperatures}} \right)$$

For this to be non-negative (as required by the Second Law of Thermodynamics) we must have

$$T > \text{Geometric Mean of Original Temperatures}$$

The first law of thermodynamics tells us that

$$T = \text{Arithmetic Mean of Original Temperatures}$$

so we can conclude that the arithmetic mean is at least as great as the geometric mean.